# Arbitrary Motion Aerodynamics Using an Aeroacoustic Approach

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**This paper describes a new unsteady aerodynamics method that uses time-domain aeroacoustic integral equations. Recent advances in theoretical aeroacoustics permit the development of general unsteady aerodynamics methods. The effects of thickness, compressibility, and arbitrary motions may be calculated for subsonic and supersonic flows. Most linearized unsteady aerodynamics methods today are limited to zero-thickness airfoils and sinusoidal motions. The method outlined in this report has applications to super-maneuverable aircraft and rotating blades. Panel methods for steady aerodynamics are widely used in the aircraft industry for steady flow, and the present method should permit equally general configurations to be solved in unsteady motion using a time-stepping procedure.**

## Nomenclature

$c$ = speed of sound
$f(\bar{x}, t)$ = function describing body surface (before linearization)
$M$ = Mach number
$M_r$ = Mach number in radiation direction, $\bar{v} \cdot \hat{r}/c$
$\hat{n}$ = unit normal vector on body surface, $\nabla f/|\nabla f|$
$p'$ = pressure perturbation
$P_{ij}$ = compressive stress tensor
$r$ = $|\bar{r}|$
$\bar{r}$ = vector distance between source and observer, $\bar{x} - \bar{y}(\tau)$
$\hat{r}$ = $\bar{r}/r$
$t$ = observer time
$T_{ij}$ = Lighthill's stress tensor, $\rho u_i u_j + P_{ij} - c^2(\rho - \rho_0)\delta_{ij}$
$\bar{u}$ = fluid velocity
$\bar{u}'$ = fluid velocity perturbation
$\bar{u}_1$ = irrotational component of velocity field
$\bar{u}_2$ = solenoidal (vortical) component of velocity field
$u_n$ = $\bar{u}' \cdot \hat{n}$
$\bar{v}$ = velocity of body surface
$v_n$ = $\bar{v} \cdot \hat{n}$
$v_r$ = $\bar{v} \cdot \hat{r}$
$\bar{x}$ = observer location in frame fixed to undisturbed fluid
$\bar{y}(\tau)$ = source location (in motion)
$\beta^2$ = $1 - M^2$
$\delta(f)$ = Dirac delta function
$\delta_{ij}$ = Kronecker delta
$\rho$ = fluid density
$\rho'$ = fluid density perturbation, $\rho - \rho_0$
$\rho_0$ = density of undisturbed fluid
$\tau$ = $t - r/c$, retarded time
$\bar{\omega}$ = $\nabla \times \bar{u}'$, vorticity
ret = expression is evaluated at retarded time, $\tau = t - r/c$
$\square^2$ = wave operator
[ ] = jump in quantity across body surface

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## Introduction

TO develop highly-maneuverable aircraft one must be able to predict time-accurate aerodynamics for rapid maneuvers, including control surface deflections and stall flutter. This requires methods capable of predicting loads for more than rectilinear or harmonic flight motions. For some flight conditions nonlinear prediction methods will be required; but, in many cases, linear arbitrary-motion methods will be sufficient especially in the early stages of design. At the present time, nonlinear methods are too costly to be used routinely in a time-accurate mode during the design process. In addition to highly maneuverable aircraft, the aerodynamics of advanced helicopters and propellers cannot be adequately predicted using methods restricted to harmonic motions.

The present paper describes a compressible arbitrary motion aerodynamic (CAMA) method based upon aeroacoustic integral equations. Classical airfoil aerodynamics, unsteady aerodynamics, and propeller aerodynamics are limited to rectilinear, harmonic, and helical motions, respectively. Arbitrary motion aerodynamics refers to methods for predicting surface loads that are not restricted to any particular type of motion. The present method is a time-domain formulation, which is necessary for completely arbitrary motions. In addition, a time-domain formulation is a prerequisite to including nonlinear effects. This article addresses primarily the linear aspects. Additional problems associated with nonlinear fluid dynamics (e.g., transonic flow) have not been modeled using the approach outlined here.

Examples of aircraft and propeller dynamic behavior that are often inadequately predicted during the design process are sub- or super-harmonic instabilities, limit cycle oscillations, single degree of freedom flutter, control system ineffectiveness, and battle-damage effects. The importance of these effects increases with higher performance aircraft, lighter more flexible structures, and complex active control systems. To accommodate these, the designers' tools must become more versatile and accurate. The arbitrary motion aerodynamics method described herein responds to this challenge by permitting the calculation of time-accurate surface pressures on arbitrary bodies in arbitrary motions. In addition, because the integral equations are derived from first principles, the proper role of the nonlinear aerodynamic terms is readily apparent.

In addition to being limited to particular types of motion, current unsteady aerodynamics methods become increasingly inaccurate as the reduced-frequency becomes large at Mach numbers approaching unity. Also, flutter speed must be determined through an iterative process. An arbitrary motion aerodynamics method would permit the direct calculation of flutter speeds. The method described here will be valid for frequencies increasing without limit, and the time stepping nature of the solution technique allows the treatment of systems with mechanical nonlinearities.

## The Ffowcs Williams-Hawkings Equation

The origin of the aeroacoustic approach used herein can be traced to the Ffowcs Williams-Hawkings (FW-H) equation[1]

$$c^2\Box^2(\rho-\rho_0)=\frac{\partial^2 T_{ij}}{\partial x_i \partial x_j} - \frac{\partial}{\partial x_i}\left(P_{ij}\frac{\partial f}{\partial x_i}\delta(f)\right)+\frac{\partial}{\partial t}\left(\rho_0 v_i \frac{\partial f}{\partial x_i}\delta(f)\right) \tag{1}$$

where

$$\Box^2=\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x_i^2}$$

is the wave operator and

$$T_{ij}=\rho u_i u_j + P_{ij} - c^2(\rho-\rho_0)$$

is Lighthill's stress tensor. This equation is a combination of the complete nonlinear inhomogeneous conservation of mass and momentum equations. Generalized function theory is extremely useful in deriving the above equation. Reference 2 is a recent text on the subject and includes classroom lecture notes by Farassat. It should also be pointed out that viscous effects are included in the compressive stress tensor $P_{ij}$. The conservation equations are inhomogeneous because the fluid is assumed to occupy both the interior and exterior of arbitrary bodies. The inhomogeneous terms represent mass and momentum sources that will be used to satisfy the boundary conditions. The above equation is obtained by differentiating the inhomogeneous mass conservation equation with respect to $t$ and the momentum conservation equation with respect to $x_i$. The latter is then subtracted from the former and rearranged such that only the wave operator is on the left-hand side. Writing the equations in this manner is referred to as the "acoustic analogy approach," which was pioneered by Lighthill.[3] However, Lighthill was concerned with the noise due to turbulent jets in regions without solid surfaces and therefore neglected the last two terms in the above equation. Acousticians frequently use the above form of the equation to model aeroacoustic noise.

If the entire right-hand side of the above equation were known, then one could presumably predict the radiated noise. However, these terms contain unknowns that must be determined before the noise can be predicted. Recently, computational fluid dynamics methods have been used to predict Lighthill's stress tensor.[4] In the past, however, classical methods or experiments were used to determine the forcing functions, and then the above theory was used to predict the noise. A more appealing approach, especially for linear theory, is to use Eq. (1) for both the aerodynamics and the acoustics. Both fields are governed by Eq. (1) and recent research[5-8] has shown that this approach is possible. In addition, the nonlinear terms are included in Eq. (1). Therefore, if one wanted to include them in the aerodynamic and/or acoustic method, their proper roles are readily apparent.

Although Eq. 1 is commonly referred to as the FW-H equation, there is a more general form that was not explicitly presented in the original paper. In Eq. (1), the flow inside the body is assumed to be quiescent. For some applications, it may be useful not to make this assumption. The equation then becomes

$$c^2\Box^2(\rho-\rho_0)=\frac{\partial^2 T_{ij}}{\partial x_i \partial x_j} - \frac{\partial}{\partial x_i}\left[P_{ij}-\rho u_i(u_j-v_j)\right]\delta(f)\frac{\partial f}{\partial x_i} + \frac{\partial}{\partial t}\left[\rho(u_i-v_i)\right]\delta(f)\frac{\partial f}{\partial x_i}$$

where the brackets indicate the jump in a flow quantity across the body surface. Of course, one should be careful not to infer too much from the above equation, it cannot contain as much information as the original four equations and four unknowns, as is shown later. The linearized, inviscid FW-H equation is

$$\Box^2 p' = -\frac{\partial}{\partial x_i}[p']\delta(f)\frac{\partial f}{\partial x_i}+\frac{\partial}{\partial t}\rho_0[u_i']\delta(f)\frac{\partial f}{\partial x_i} \tag{2}$$

where $p'=c^2(\rho-\rho_0)=c^2\rho'$ and $[p']=p_2'-p_1'$, denoting the jump in pressure across the surface, and $[u_i']$ the jump in the velocity across the surface. Again, Eq. (2) does not assume quiescent flow inside the body. The primes denote perturbational quantities, and, since only the linear equations will be discussed herein, the primes will no longer be shown.

## Farassat's Solution to The FW-H Equation

Farassat[9,10] has derived several useful integral equation versions of the previous equation using Green's functions. One of the most well-known is

$$4\pi p(\bar{x},t)=\frac{\partial}{\partial t}\iint\left\{\frac{\rho_0 c[u_n]+[p]\cos\theta}{cr|1-M_r|}\right\}_{\text{ret}}\mathrm{d}S + \iint\left\{\frac{[p]\cos\theta}{r^2|1-M_r|}\right\}_{\text{ret}}\mathrm{d}S \tag{3}$$

where ret signifies retarded time. That is, the integrands are evaluated at the emission time $\tau=t-r/c$ instead of the reception time $t$. Once again, this is not usually written explicitly with jump conditions. Instead, most researchers assume the internal flow to be quiescent, which yields

$$4\pi p(\bar{x},t)=\frac{\partial}{\partial t}\iint\left\{\frac{\rho_0 c v_n+p\cos\theta}{cr|1-M_r|}\right\}_{\text{ret}}\mathrm{d}S + \iint\left\{\frac{p\cos\theta}{r^2|1-M_r|}\right\}_{\text{ret}}\mathrm{d}S \tag{4}$$

where now the $p$ under the integral is the same as the $p$ on the left hand side (when the singularity is properly accounted for) and the boundary condition $u_n=v_n$ (no normal flow) has been applied.

The above equation is only one example of the integral form of the differential equation. Other forms are available and should not be ruled out for practical applications. Reference 10 summarizes many of the existing integral equations for aeroacoustics. In particular, Eqs. (15b) and (17b) of Ref. 10 are equivalent to Eq. (4) here; however, the integration is carried out over the intersection of the "collapsing" sphere and the body surface, instead of over the body surface in retarded time. Even though these other forms are sometimes difficult to use because one must determine these intersections, they may prove useful in certain aerodynamic applications. In addition, Refs. 5 and 11 present Eq. (4) with the time derivative brought inside the integral.

Equation (4) appears to be very useful since the pressure is the only unknown. The velocity is known from the boundary conditions. However, the Eq. (4) by itself is of limited utility for aerodynamics, since the internal flow has been specified and there are no means of accounting for the wake development. Consequently, there is no circulation or lift. This can be easily shown by letting the thickness of the body go to zero, which yields

$$4\pi p(\bar{x}, t) = \frac{\partial}{\partial x_i} \iint \left\{ \frac{[p] n_1}{r|1 - M_r|} \right\}_{\text{ret}} \mathrm{d}S$$

which is one equation for two unknowns ($p$ and $[p]$). Also, there are no means of specifying the boundary conditions. This occurs because the pressure is uncoupled from the vortical part of the velocity field (in linearized theory). This can be shown using the well-known splitting theorem,[12] where the velocity field is split into irrotational and solenoidal (vortical) parts, i.e.,

$$\bar{u} = \bar{u}_1 + \bar{u}_2$$

where

$$\nabla \times \bar{u}_1 \equiv 0 \quad \text{and} \quad \nabla \cdot \bar{u}_2 \equiv 0$$

It can also be shown[12] that the homogeneous conservation laws become

$$\frac{1}{c^2}\frac{\partial p}{\partial t} + \rho_0 \nabla \cdot \bar{u}_1 = 0 \quad \rho_0 \frac{\partial \bar{u}_1}{\partial t} + \nabla p = 0 \quad \frac{\partial \bar{u}_2}{\partial t} = 0$$

which shows that the vortical velocity field is independent of the pressure (or acceleration potential) field. Only in nonlinear theory are the governing equations for the acoustic pressure coupled to the vortical or solenoidal velocity. Therefore, the vortical velocity can only be coupled to the pressure via the boundary conditions. This means that while specifying the boundary conditions in the FW-H equation does completely specify the thickness problem, it is not enough to solve the general lifting problem. Incidentally, in supersonic flows where the vortical field is unimportant (such as those with supersonic leading and trailing edges), the solution can be adequately represented by "thickness" or irrotational terms. In this case, Eq. (4) is a complete specification of the problem.

In a velocity potential formulation, such as those used in panel methods, one must distribute doublets over the body surface and the wake. Since the pressure jump across a wake is zero, one might assume that Eq. (4) is as general as the Green's theorem formulation of the velocity potential, without requiring any integration over the wake. However, in the above formulation, the normal-wash contributions are not included because they are due to the vortical velocity field. Therefore, the effects due to the wake (and consequently lift) are not included.

## Compressible Arbitrary Motion Aerodynamics

Even though Eq. (4) is not enough in itself to develop an arbitrary motions aerodynamics method, there are several options that permit its use in such a method. One rather novel solution technique would be to use Eq. (3) to solve for the flowfield in both regions simultaneously, i.e., inside and outside the body. A second technique might be to use the splitting theorem mentioned earlier. This would not change the operator on the pressure but would effect the application of boundary conditions, since there would be another unknown (the normal component of the solenoidal velocity). This additional unknown would appear as another "thickness" term in the integral equation (thickness is, of course, a misnomer in this case). However, in both of these approaches, one obtains more unknowns than equations. Thus one must use either the equations governing the internal flow or one of the conservation laws, in addition to the integral equation for the pressure.

As an aside, it should be pointed out that the method described in Refs. 5 and 6 for predicting lift using Eq. (4) can be shown to be related to the two procedures just mentioned. However, instead of solving for the additional unknown that appears (i.e, the vortical velocity or the internal flow), these terms are approximated using a "conditioning" procedure. This conditioning was prompted by a recommendation of van Holten[13] that only the leading edge be placed at an angle of attack when using acceleration potential methods. The vortical component of the velocity would have such an effect, but it must be determined as part of the solution in a general prediction method. Even though this technique produced good results for uncambered, symmetrical blades and wings, it would be very difficult to apply to arbitrary bodies.

The technique that will be described in this paper is an alternate approach, closely related to classical unsteady aerodynamics methods. It utilizes the linearized conservation of momentum equation as the second equation. Its validity is easily demonstrated because of its relationship to well-known unsteady aerodynamics methods. The linearized inhomogeneous conservation of momentum equation is

$$\rho_0 \frac{\partial \bar{u}}{\partial t} = -\nabla p + [p] \nabla f \delta(f) \tag{5}$$

which for a field point off the surface is

$$\rho_0 \frac{\partial \bar{u}}{\partial t} = -\nabla p$$

or

$$\rho_0 \bar{u}(\bar{x}, t) = -\int_{-\infty}^{t} \nabla p(\bar{x}, t') \, \mathrm{d}t' \tag{6}$$

where $t'$ is the integration variable. This equation can be combined with Eq. (4) to eliminate the pressure $p$ thus giving an equation relating the fluid velocity $u$ to the surface quantities: pressure jump $[p]$ and normal velocity $v_n$. The resulting equation describes the relation between an arbitrary surface loading and the downwash unlike Eq. (4). Most importantly though, it accounts for the growth of a wake (which will simply be the body at previous times) and the generation of vorticity. Therefore, for a wing, the wake will lie in the plane directly behind the wing; and for a propeller, the wake will be formed by the helical path of the blade.

In order to illustrate the origin of vorticity in the aeroacoustic approach, take the curl of Eq. (5) to get

$$\rho_0 \frac{\partial \bar{\omega}}{\partial t} = \nabla \times \{[p] \nabla f \delta(f)\}$$

where

$$\bar{\omega} = \nabla \times \bar{u}$$

which relates the rate of change of vorticity to the momentum sources on the surface; that is, vorticity is only generated at the surface. However, this vorticity must remain in its place of origin, since there is no vorticity convection or diffusion included in these equations. Von Kårmån and Burgers presented similar relations for an incompressible fluid in Ref. 14, where the fluid is assumed to have an arbitrary distribution of force. These forces, analogous to the source terms above, are the source or vorticity. Reference 14 also discusses (in different terminology) splitting the velocity of a perfect fluid into solenoidal and irrotational parts. The previous relation also shows that for a constant pressure panel method, the vorticity is generated only at the panel edges, since the surface-pressure gradients are zero everywhere else. This is one illustration of how the method is related to the vortex lattice method.

Combining Eqs. (4) and (6) gives

$$4\pi\rho_0\bar{u}(\bar{x},t)=\nabla\iint\limits_S\left\{\frac{\rho_0 c v_n+p\cos\theta}{cr|1-M_r|}\right\}_{\mathrm{ret}}\mathrm{d}S$$

$$-\nabla\int_{-\infty}^{t}\iint\limits_S\left\{\frac{p\cos\theta}{r^2|1-M_r|}\right\}_{\mathrm{ret}}\mathrm{d}S\,\mathrm{d}t' \qquad (7)$$

Applying the boundary condition

$$v_n(\bar{x},t)=u_n(\bar{x},t)$$

gives

$$4\pi\rho_0 v_n(\bar{x},t)=-\frac{\partial}{\partial n_x}\iint\limits_S\left\{\frac{\rho_0 c v_n+p\cos\theta}{cr|1-M_r|}\right\}_{\mathrm{ret}}\mathrm{d}S$$

$$-\frac{\partial}{\partial n_x}\int_{-\infty}^{t}\iint\limits_S\left\{\frac{p\cos\theta}{r^2|1-M_r|}\right\}_{\mathrm{ret}}\mathrm{d}S\,\mathrm{d}t' \qquad (8)$$

where $\hat{n}_x$ is the surface normal at $\bar{x}$, the "observer" position (which is distinguished from $\hat{n}_y$ the normal at $\bar{y}$, the "source" position). The above equation relates the time-history of the surface pressure and normal velocity of an arbitrary surface to the fluid velocity at a control point. Equation (8) was presented by Farassat in Ref. 11 in a discussion of how the aeroacoustic equations relate to classical unsteady aerodynamics equations. It has not been proposed as the basis of an arbitrary motions aerodynamics method before. Incidentally, once the surface pressure is known, Eq. (7) can be used to predict the velocity at any point in the flowfield.

Notice that in this formulation the time history of the body's motion accounts for the wake effects of potential theory. The pressure jump is zero across a wake, but the effect from a pressure jump at that location (when the body was located there) lingers on after the body has moved. In other words, the pressure loading only exists for a certain amount of time, but after it is gone, there remains a vortical velocity field. This is illustrated in Fig. 1. An S-3A aircraft is shown at three different times during a maneuver. The current time is $t$ and two previous times ($\tau_1$ and $\tau_2$) are also shown. The impulsive pressures that existed at $\tau_1$ and $\tau_2$ continue to affect the flowfield. As mentioned earlier, the governing equations for the pressure are decoupled from the vortical field; therefore, only in the boundary and initial conditions can they be coupled. This effect accounts for the unsteady shedding of a vortex sheet in a very natural manner.

Equations (7) and (8) are unique because thickness effects, compressibility, and arbitrary motions can be modeled by them. These equations should permit the development of unsteady aerodynamics methods that are very general compared to existing methods. Of course, Eq. (8) must be regularized before it can be used in a numerical method, which means the singularity must be properly treated. The treatment of this singularity was discussed in Ref. 15.

If thickness effects are neglected, Eq. (8) becomes

$$4\pi\rho_0 v_n(\bar{x},t)=-\frac{\partial}{\partial n_x}\iint\limits_S\left\{\frac{p\cos\theta}{cr|1-M_r|}\right\}_{\mathrm{ret}}\mathrm{d}S$$

$$-\frac{\partial}{\partial n_x}\int_{-\infty}^{t}\iint\limits_S\left\{\frac{p\cos\theta}{r^2|1-M_r|}\right\}_{\mathrm{ret}}\mathrm{d}S\,\mathrm{d}t'$$

which can be approximated by

$$v_n(\bar{x},t)=\frac{-1}{4\pi\rho_0}\frac{\partial}{\partial n_x}\int_{-\infty}^{t}\iint\limits_S\left\{\frac{\beta^2[p]\cos\theta}{r^2|1-M_r|^3}\right\}_{\mathrm{ret}}\mathrm{d}S\,\mathrm{d}t' \qquad (9)$$

which is derived using a form of Eq. (4) with the time derivative taken inside the integral (see Ref. 5) and assuming the acceleration of the body is small. The compactness of these equations is surprising considering their generality. Included in these equations is the incompressible, steady Biot-Savart law. They are valid for completely arbitrary motions at subsonic and supersonic speeds. Given the loading on a lifting surface anywhere in space and time, Eq. (9) will predict the velocity field due to that loading. Inversely, which is how the above equation can be used, given the velocity of a body, Eq. (9) will predict its time-accurate surface loading. The previous equation is closely related to those of Dat,[16] Runyan and Tai,[17] and Kussner.[18] Kussner, however, used a Lorentz transformation which would only be useful for a body whose mean motion is rectilinear. Kussner's formulation has formed the foundation of unsteady aerodynamics for many years. It is the basis for the kernel function approach[19] and the doublet lattice method,[20] both of which assume a rectilinear mean motion and a harmonic perturbational velocity.

## Numerical Scheme

The most effective numerical scheme for solving the previous equations is still open to question. This section presents one possible approach. The method will be applied to Eq. (9); but the procedure would also apply to Eq. (8), which includes thickness effects. In aerodynamic panel methods for steady flow, the surface of the body is approximated by a finite number of discrete panels. The dependent variable (usually velocity potential) is assumed to vary in some prescribed manner over each panel (e.g., constant, bilinear, etc.). This procedure could also be used for the equations described in this report; that is, the body could be approximated by panels of constant pressure.

In the present method the pressure must also be discretized as a function of time. For example, if the pressure distribution is assumed constant over a set of discrete emission time intervals, i.e.,

$$\begin{aligned}
[p(\bar{y},\tau)]&=[p(\bar{y})]^1, && 0<\tau<\tau_1\\
&=[p(\bar{y})]^2, && \tau_1<\tau<\tau_2\\
&\ \vdots && \ \vdots\\
&=[p(\bar{y})]^{NT}, && \tau_{NT-1}<\tau<\tau_{NT}
\end{aligned}$$

then Eq. (9) could be written

$$v_n(\bar{x},t)_i=\sum_{k=1}^{NT}\frac{-1}{4\pi\rho_0}\frac{\partial}{\partial n_x}\int_{\infty}^{t}\iint\limits_S\left\{\frac{\beta^2[p]^k\cos\theta}{r^2|1-M|^2}\right\}_{\mathrm{ret}}\mathrm{d}S\,\mathrm{d}t'$$

where the superscript $k$ indicates emission time (not exponentiation) and $NT$ is the total number of time steps. Now for each emission time, the surface can be approximated by a

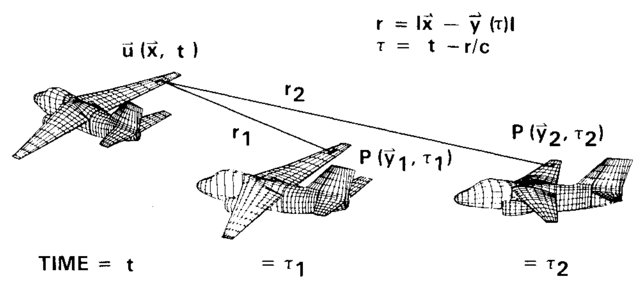


**Fig. 1 S-3A aircraft at various positions in a maneuver, illustrating time dependant solution procedure.**

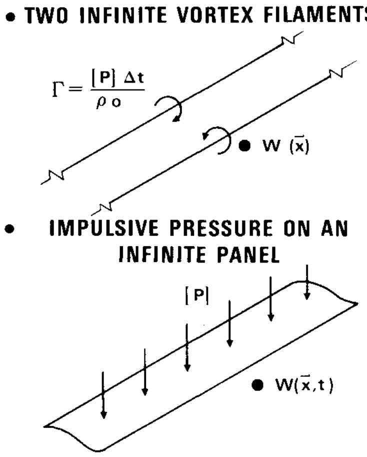


Fig. 2 Two infinite vortices are equivalent to impulsive pressure applied to infinite flat plate in CAMA.

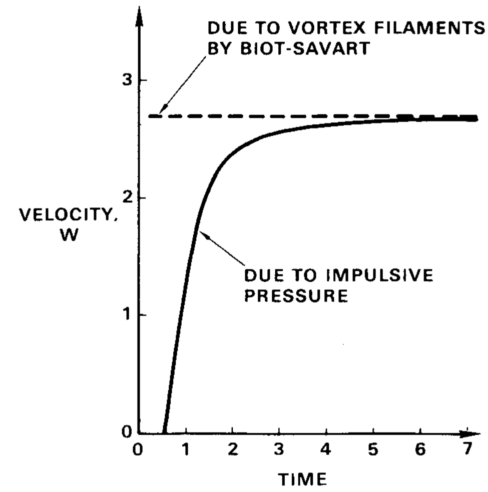


Fig. 3 Velocity induced by two infinite vortices (Biot-Savart) and by CAMA. The control point is in the same plane as the two vortices and is $\Delta x/2$ from the nearest vortex.

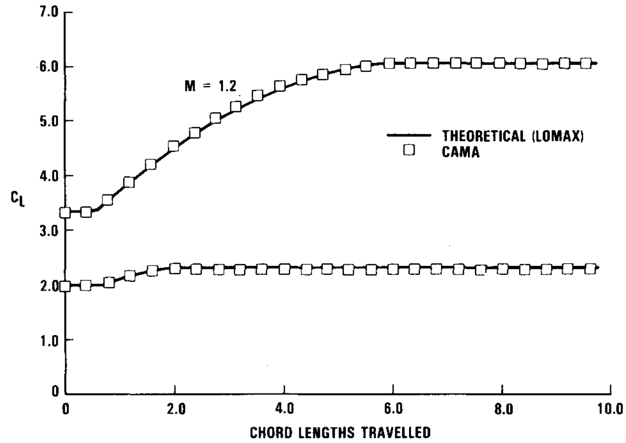


Fig. 4 Lift coefficient on an impulsively-started plunging flat plate at Mach 1.2 and 2.0. Plunge velocity equals forward speed.

finite number of constant pressure panels giving

$$v_n(\bar{x},t)_i = \sum_{k=1}^{NT}\sum_{j=1}^{NP} A_{ij}^k [p]_j^k$$

$$A_{ij}^k = \frac{-1}{4\pi\rho_0}\frac{\partial}{\partial n_x}\int_{-\infty}^{t}\iint_S \left\{\frac{\beta^2\cos\theta}{r^2|1-M_r|^2}\right\}_{\mathrm{ret}} \mathrm{d}S\,\mathrm{d}t'$$

where $NP$ is the total number of panels. Evaluating this at the control points (the centroid of each panel) gives a system of linear algebraic equations

$$(v_{n_i}) = (A_{ij}^1)([p]_j^1) + (A_{ij}^2)([p]_j^2) + \cdots + (A_{ij}^k)([p]_j^k)$$

and rewriting gives

$$([p]_j^k) = (A_{ij}^k)^{-1} \times \left\{(v_{n_i}) - (A_{ij}^1)([p]_j^1) - (A_{ij}^2)([p]_j^2) - \cdots\right\} \qquad (10)$$

From causality, during the first time step only, the first matrix will influence the solution, thus

$$([p]_j^1) = (A_{ij}^1)^{-1}(v_{n_i})$$

which combined with an appropriate regularization procedure (as outlined in Ref. 15) will predict the pressure during the first time step. For subsequent time steps, Eq. (10) can be used, which contains only known information on the right-hand side (i.e., the boundary condition and the pressure at previous time steps).

One must still determine the most appropriate variation of pressure over a panel. In fact, instead of prescribing some variation over the panel, one could assume the entire load exists along a line on each panel as in the doublet lattice method.[20] One could also concentrate the force at the centroid of each element as in Ref. 21. This would be less accurate but would not require any quadrature.

It may be useful to make some general comments concerning the approach outlined here. First of all, the matrix of influence coefficients at the latest time step is the only one that must be inverted as shown in Eq. (10). In addition, this matrix will be either banded or diagonal due to the effect of the domain of influence. In effect, the pressure corresponding to the latest emission time has not propagated very far from its point of origin. Therefore, these panels will only influence themselves and, possibly, their immediate neighbors. This situation corresponds to piston theory,[22] which is often used in unsteady aerodynamics. Piston theory gives a linear relation between the pressure and the velocity, i.e., $[p] = 2\rho_0 c v_n$. If the latest matrix is a diagonal and the effects from the panel edges do not affect the self-influence terms, then the elements of the matrix will be given exactly by this relation.

When the initial effects are relatively removed from the latest time step, the initial transients will have a minor effect and, in a numerical scheme, could be ignored. Thus, as the time-marching scheme progresses and the elements of the early matrices become small, the initial effects may be neglected. This is analogous to truncating the wake in steady panel methods and will reduce the computer storage required by the program. Therefore, a finite amount of computer storage will be required even for relatively large values of the observer time.

The method described is a time-accurate pressure or acceleration-potential formulation, based upon the linearized Euler equations. It is believed that this method will automatically satisfy the Kutta condition because vorticity is generated automatically. The wake is explicitly unloaded, and the initial conditions (or starting vortex) are included. The method is the compressible equivalent of an unsteady vortex lattice method that sheds vortex elements.

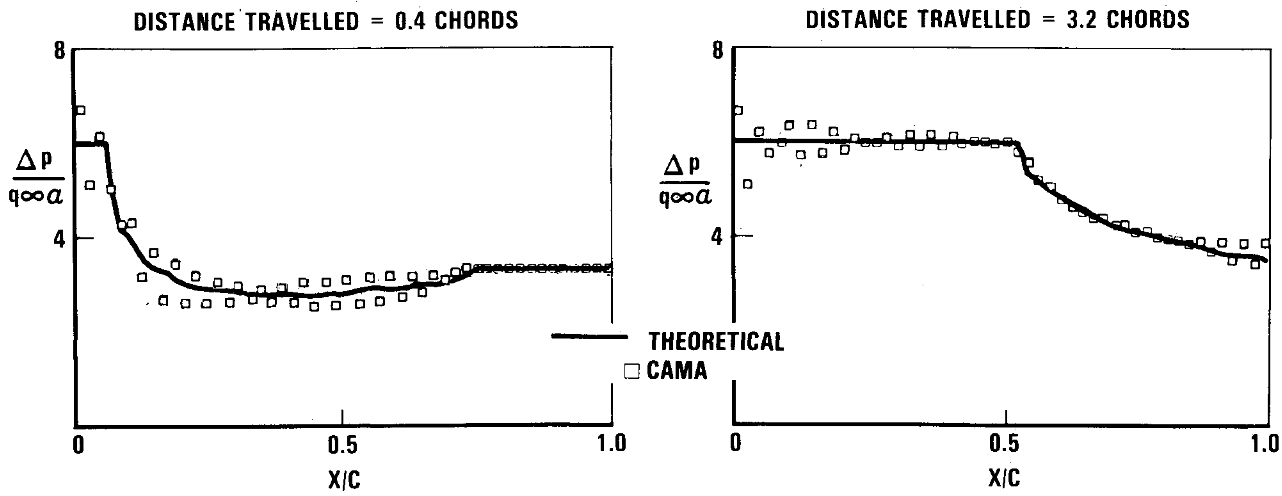


Fig. 5 Pressure jump across plunging flat plate at Mach = 1.2. ($U$*time/chord = 0.4 and 3.2).

## Results

In order to test the described theory, a computer program called CAMA (Compressible Arbitrary Motion Aerodynamics) has been developed. This program has been used to study two sample problems. In both of these problems, the influence coefficients (including the effects of retarded time) were obtained analytically using the collapsing sphere concept described in Ref. 10.

The first example illustrates how the aeroacoustic formulation (Eq. 9) compares to the Biot-Savart law. Shown in Fig. 2 are two infinite vortices separated a distance $\Delta x$. Using the Biot-Savart law, one can show that these induce a velocity $w$ at the point $x$ ($\Delta x/2$ from the nearest vortex) as shown in Fig. 3. The induced velocity is made nondimensional by dividing it by $(\Gamma/4\pi\Delta x)$. This is a steady-state result.

The CAMA formulation predicts the induced velocity due to an impulse pressure applied to a surface (Fig. 2); this is a time-dependent response. The circulation is related to the impulse pressure through $\Gamma = [p]\Delta t/\rho_0$. The induced velocity in this formulation (Fig. 3) grows with time to asymptotically match the induced velocity given by the Biot-Savart law. The time has been made nondimensional by dividing it by $(\Delta x/c)$. The equation presented above (Eq. 9) is essentially a compressible version of the Biot-Savart law. Another important phenomenon predicted by CAMA is the time-lag of the response. Due to the finite speed of propagation of the disturbance, the field point does not sense that a pressure has been applied to the panel until the disturbance reaches it. Thus, the induced velocity at point $x$ does not start increasing until a short time after the pressure is applied.

The second example is an impulsively-started plunging flat plate initially immersed in an infinite flow tangent to the plate. This will be compared to the classical results of indicial aerodynamics. This was thought to be a good test case because of the rapid changes in the flowfield that occur and the importance of time-accuracy. Figure 4 shows the lift coefficient of a flat plate at $M = 1.2$ and 2.0 as a function of time. The predictions show excellent agreement with the theoretical predictions of Lomax.[23] Figure 5 shows chord-wise pressure distributions for the flat plate at a Mach number of 1.2 and two different times ($U^*t$/chord = 0.4 and 3.2). The pressure is seen to oscillate about the theoretical curves of Lomax. The integrated values compare very well as shown on the previous figure.

## Conclusion

This paper describes a linearized, compressible, arbitrary motions aerodynamics method that uses aeroacoustic integral equations. The method is essentially a time-domain acceleration-potential (or pressure) method. The majority of existing unsteady aerodynamics methods assume harmonic motion, which is inadequate to predict many preliminary design problems, as discussed in the introduction. This research represents the latest efforts by the Lockheed-California Company in the development of general arbitrary motions aerodynamics method. Previous efforts have concentrated on incompressible methods, including pilot computer codes.[24]

Even though transonic and viscous effects are very important in aeroelasticity,[25] it is believed that the method described will be useful because of its efficiency when compared to full nonlinear methods. The aeroacoustic equations use the Green's function technique to convert the three-dimensional problem into a two-dimensional surface integration. Its role can be compared to that of steady aerodynamic panel methods. Most panel methods use linearized forms of the full-potential equation. The method described in this report is based upon the linearized Euler equations. Very sophisticated codes are currently available for solving the full-potential, Euler and Navier-Stokes equations; however, these codes are not yet efficient enough to be routinely used by designers to predict unsteady or arbitrary motion effects. Therefore, just as panel methods are heavily used for steady flows, especially for preliminary design, the method described in this report could be developed into a general arbitrary motion panel method and would be very useful for preliminary design.

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